Abstract. This paper analyses the role of inflation in economies with endogenous growth and congestion in public services. Optimal policy rules are derived for public services and investment. The other findings are as follows. Monetary policy should maximize economic growth. The more inefficient the public sector is, the higher the growth-maximizing inflation rate is. If a currency union accepts a new member with an inefficient public sector, this will boost inflation in the union and decrease growth and welfare in all member economies of the union.


Keywords: Growth; inflation; congestion; public investment.

1. INTRODUCTION

This paper considers the effects of inflation on growth and welfare. Because the optimal monetary policy is crucially dependent on what fiscal policy instruments are available for use in conjunction with it, the entire public sector must be modelled at the same time as monetary instruments. In this unified approach, the public sector produces services for both households and firms, and finances these by taxes and seigniorage.

Many papers that consider the relationship between inflation and growth assume some form of non-distorting taxation. In such a case, however, there is no explanation of inflation: it is not optimal for the government to impose seigniorage at all, because the same revenue is more easily collected by lump-sum taxation. On the other hand, the papers that examine fiscal policy in the framework of economic growth usually ignore inflationary financing.

1. This is very often done through the assumption that the government can pay lump-sum transfers to (or extract lump-sum taxes from) private agents. Cf. Zhang (1996), Huo (1997), Pecorino (1997), and Chang et al. (2000).

Palokangas (1997) considers inflation and growth in a model of public finance, ignoring the government’s spending on services and investment. He assumes that taxes and seigniorage are needed only to finance interest on public debt. Ferreira (1999) examines inflation and growth in an OLG model in which the young save fiat money in order to spend it after retirement and public spending is assumed to increase productivity. He shows that growth maximization can never be optimal and that in general a higher inflation rate yields a higher growth rate. Our study extends this literature so that public services and public investment are financed by seigniorage and distorting taxation. However, we ignore the life-cycle aspect, according to which inflation redistributes wealth between generations, and concentrate on the effect of inflation on efficiency. We consider the dynamic equilibrium of the economy in the framework of a representative agent which consumes, produces, holds money reserves and uses government services.

In this study, a key feature of the analysis is the central role assigned to congestion – the bigger the agent is, the more it expects to benefit from government input. As Barro and Sala-i-Martin (1995) have argued, virtually all public services are characterized by congestion. We show that a specific form of congestion is necessary for persistent growth, and that growth maximization is optimal monetary policy for a growing economy. The paper is structured as follows. Section 2 introduces the basic assumptions of the model. Section 3 establishes the choices of consumption, production, investment and cash balances. In Section 4, optimal public policy is constructed for the circumstances in which government services are input to private production. This basic model is then extended to the case of government investment in Section 5. The optimal monetary policy is specified in Section 6.

2. THE SETTING

2.1. Production and consumption

We aggregate all goods in the economy into a single good, the price of which is $p$. There are two assets, money and capital. There is a fixed number $J$ of similar private agents who save, invest in capital, hold money for transaction purposes and produce goods from capital.

Government services benefit both production and consumption. Private agents are able to transfer resources between taxed and non-taxed sectors of the economy.$^3$ Output in the taxed sector, $Y$, is a concave and linearly homogeneous function $F$ of government input $G$, capital stock $K$ and output $Y$.

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3. This assumption is needed to produce a distortion in the public sector, which gives the government the incentive to use seigniorage.
in the non-taxed sector, \( Z \), as follows:

\[
Y = F(G, K, Z) = F(g, 1, z)K \quad F_G \equiv \partial F/\partial G > 0 \quad F_K \equiv \partial F/\partial K > 0 \\
F_Z \equiv \partial F/\partial Z < 0 \quad z \equiv Z/K \quad g \equiv G/K
\]  

(1)

The intertemporal utility function is given by

\[
\int_0^\infty U(C, G)e^{-\rho t} dt = \int_0^\infty U(c, g)K^{1-\sigma} e^{-\rho t} dt \quad \text{with}
\]

\[
U(c, g) \equiv (1 - \sigma)^{-1}[(1 - \alpha)c^{1-\sigma} + \alpha g^{1-\sigma}] \quad \text{and} \quad c \equiv C/K
\]

where \( t \) is time, \( C \) consumption, \( \rho > 0 \) the rate of time preference (constant), \( \alpha \in [0, 1) \) the relative weight of government services (constant), and \( \sigma \in (0, 1) \cup (1, \infty) \) the inverse of the intertemporal elasticity of substitution (constant). We denote the marginal rate of substitution between consumption and government services when the level of instantaneous utility, \( U \), is kept constant as follows:

\[
\partial C / \partial G \bigg|_{U \text{ constant}} = \frac{\alpha}{1 - \alpha} \left( \frac{C}{G} \right)^\sigma = \frac{\alpha}{1 - \alpha} \left( \frac{c}{g} \right)^\sigma > 0
\]

(3)

### 2.2. Congestion

Congestion results from the existence of many agents. As distinct from aggregate output \( Y \) and aggregate capital \( K \), we denote a single agent’s output and capital by \( y \) and \( k \). With congestion, a single agent assumes it will get the more services \( G \) from government input \( G \) the larger its share \( k/K \) of aggregate capital stock \( K \). In line with Fischer and Turnovsky (1998), we specify this as follows:

\[
\hat{G} \equiv (k/K)^\delta G \quad \text{with} \quad 0 < \delta \leq 1
\]

(4)

where \( \delta \) is a parameter. When congestion is proportional, \( \delta = 1 \), the agent assumes that it receives public services \( \hat{G} \) in direct proportion to its capital stock \( k \). When congestion is partial, \( 0 < \delta < 1 \), the agent assumes that it receives less \( \hat{G} \) than in proportion to \( k \). The study ignores the case of no congestion, \( \delta = 0 \), where government input \( G \) is a non-rival and non-excludable public good available equally to each agent.

The agent takes government input \( G \), aggregate capital stock \( K \) and \( g \equiv G/K \) as given. It perceives the true production function (1) and the true utility function (2), but so that macroeconomic variables \( G, Y \) and \( K \) are replaced by microeconomic variables \( \hat{G}, y \) and \( k \). Because the agents are similar, the

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4. Function (1) can also be interpreted as a transformation curve between the taxed and non-taxed sectors and \( F_Z < 0 \) as the slope of this curve.
consumption–capital ratio \( c \) and the ratio between the output of the non-taxed sector and capital, \( z \), must be the same for the economy as a whole and for a single agent. Given (4), we then obtain a single agent’s perceived production function (5) and perceived utility function (6) as follows:

\[
\begin{align*}
    y &= F(\tilde{\gamma}, 1, z) k = F((k/K)^{\delta-1} g, 1, z) k \\
    \int_0^\infty U(ck, \tilde{G}) e^{-\rho t} dt &= \int_0^\infty U(c, (k/K)^{\delta-1} g) k^{1-\sigma} e^{-\rho t} dt
\end{align*}
\]

2.3. Transaction technology

We introduce money as an intermediary good which reduces transaction costs. Feenstra (1986) shows that under certain conditions, the approach of placing money in the utility function is equivalent to the approach we are using. One unit of transaction services is produced from one unit of the composite good. The transaction technology is the same as in Kimbrough (1986), De Gregorio (1993) and Palokangas (1997). The requirement for real transaction services, \( V \), is an increasing function of real expenditure \( ck + k \), which consists of consumption \( ck \) and investment \( k = \dot{k} \), and a non-increasing function of the real level of money balances, \( M/p \), which is the ratio of the money supply \( M \) and the price level \( p \), \( V = V(ck + \dot{k}, M/p) \). This function is also linearly homogeneous, i.e. a proportional increase in both \( ck + k \) and \( M/p \) increases \( V \) by the same proportion.

To obtain a stable demand function for money, we assume furthermore that \( V \) is strictly concave and thrice differentiable. The function \( V \) can then be transformed into the form \( V = v(m)(ck + k) \), where \( m = M/(ck + k)p \) is the money–expenditure ratio and \( v(m) \equiv V(1, m) \) is a thrice differentiable function with \( v'' > 0 \). Finally, we assume that there is a bliss point \( \overline{m} \) for the money–expenditure ratio with \( m \leq \overline{m} \) and \( v(\overline{m}) = v'(\overline{m}) = 0 \). Defining the rate of investment by \( \phi \equiv \dot{k}/k \geq 0 \), we can summarize the transaction technology as follows:

\[
\begin{align*}
    0 &\leq m \equiv M/[(c + \phi)kp] \leq \overline{m} \quad 0 \leq v(m) = V/|(c + \phi)k| < 1 \\
    v' &\leq 0 \quad v'' > 0 \quad v(\overline{m}) = v'(\overline{m}) = 0
\end{align*}
\]

3. THE AGENTS

An agent’s budget constraint is given by

\[
(1 - x)y + zk = ck + \dot{k} + iM/p + V
\]
Inflationary Financing of Government Expenditure

where \( y \) is its output in the taxed sector, \( x \) the tax rate, \( zk \) its output in the non-taxed sector, \( ck \) its consumption, \( \dot{k} \) its investment, \( M \) its money supply, \( iM/p \) the depreciation in its real balances \( M/p \) due to the inflation rate \( i \), and \( V \) its purchase of transaction services. Given (5), (7) and \( \phi \equiv \dot{k}/k \), the budget constraint (9) takes the form

\[
(1 - x)F((k/K)^{\delta-1}g, 1, z) + z = [1 + im + v(m)](c + \phi) \tag{9}
\]

An agent maximizes its utility (6) subject to its budget constraint (9) and its capital accumulation \( \phi \equiv \dot{k}/k \) by the consumption–capital ratio \( c \), the rate of investment, \( \phi \), the ratio of the output in the non-taxed sector to capital, \( z \), and the money–expenditure ratio \( m \), given the inflation rate \( i \), the tax rate \( x \), government input \( G \), the aggregate capital stock \( K \) and \( g = G/K \).

In the agent’s steady state, the ratios \( c, \phi, z \) and \( m \) are kept constant. Given the budget constraint (9), this means that the average product of capital,

\[
(1 - x)y/k + z = (1 - x)F((k/K)^{\delta-1}g, 1, z) + z
\]

and the term \( k^{\delta-1} \) are kept constant. Capital stock \( k \), output in the taxed sector, \( y \), and output in the non-taxed sector, \( zk \), are constant for \( 0 < \delta < 1 \). If \( \delta = 1 \), \( k \) is undetermined, the rate of investment \( \phi = \dot{k}/k \) can be positive and output can grow at a positive rate. This can be rephrased as:

**Proposition 1.** Persistent growth is possible only with proportional congestion \( \delta = 1 \). With partial congestion, \( 0 < \delta < 1 \), there is no such growth.

With proportional congestion, the average product of capital, \((1 - x)y/k + z\), is constant, there is no equilibrium for capital stock and capital grows indefinitely. With partial congestion, the average product of capital is decreasing, there is an equilibrium for capital stock and there is no growth in capital.

Since our purpose is to examine public policy in an economy with persistent growth, then, given Proposition 1, we can focus wholly on the case \( \delta = 1 \). Given \( \delta = 1 \), (6) and (9), the agent maximizes its utility

\[
\int_0^\infty U(c, g)k^{1-\sigma}e^{-\rho t}dt = \int_0^\infty \frac{e^{-\rho t}}{1 - \sigma}[(1 - x)c^{1-\sigma} + zg^{1-\sigma}]k^{1-\sigma}dt
\]

subject to the budget constraint \((1 - x)F(g, 1, z) + z = [1 + im + v(m)](c + \phi)\) and capital accumulation \( \phi \equiv \dot{k}/k \) by variables \( c, \phi, z \) and \( m \), given \( i, x, G, K \) and \( g \equiv G/K \). The Lagrangean of this maximization is given by

\[
\Psi = [(1 - x)c^{1-\sigma} + zg^{1-\sigma}]k^{1-\sigma}/(1 - \sigma) + \mu\phi k + \omega(1 - x)F(g, 1, z) + z - [1 + im + v(m)](c + \phi) \tag{10}
\]
where $\omega$ is a Lagrangean multiplier and variable $\mu$ evolves such that

$$\mu = \rho \mu - \partial \Psi / \partial k = (\rho - \phi)\mu - [(1 - \alpha)c^{1-\sigma} + zg^{1-\sigma}]k^{-\sigma} \quad \lim_{t \to \infty} \mu e^{-\rho t} = 0 \quad (11)$$

The maximization of the Lagrangean (10) by $\mu$ leads, by duality and by the properties of the production function (5), to the definition

$$\pi(g, x) \equiv \max_{z}[(1 - x)y/k + z] = \max_{z}[(1 - x)F(g, 1, z) + z]$$

$$\pi \equiv \partial \pi / \partial g > 0 \quad \pi_x \equiv \partial \pi / \partial x < 0 \quad \pi_{xx} \equiv \partial^2 \pi / \partial x^2 > 0 \quad (12)$$

where $\pi$ is income per unit of capital. We define the elasticity of the tax base $y$ with respect to the tax rate $x$, when capital $k$ and public service intensity $g$ are kept constant, as

$$\eta \equiv -x \pi_{xx}/\pi_x > 0 \quad (13)$$

Given (6), the maximization of the Lagrangean (10) by $m$ yields

$$v'(m) = -i \quad (14)$$

From this and (7) it follows that the money–expenditure ratio, $m$, is a function of the inflation rate $i$ only:

$$m(i) \quad m' = \frac{dm}{di} = -\frac{1}{v'} < 0 \quad \varepsilon(i) \equiv -\frac{im}{m} = \left\{ \begin{array}{ll} > 0 & \text{for } m < \bar{m} \\ = 0 & \text{for } m = \bar{m} \end{array} \right. \quad (15)$$

where $\varepsilon$ is the elasticity of the demand for money with respect to the inflation rate $i$, when expenditure $(c + \phi)kp$ is kept constant.

The maximization of the Lagrangean (10) by $c$ and $\phi$ yields

$$\partial \Psi / \partial c = (1 - \alpha)c^{-\sigma}k^{1-\sigma} - [1 + im + \nu(m)]\omega = 0$$

$$\partial \Psi / \partial \phi = \mu k - [b + 1 + im + \nu(m)]\omega = 0 \quad (16)$$

The agent's output $y$ and consumption $ck$ are now in fixed proportion to capital stock $k$, which is the agent's only state variable. Consequently, the system jumps immediately to the steady state and there are no transitional dynamics. Given the first-order conditions (16), we obtain $\mu = (1 + im + \nu)\omega = (1 - \alpha)c^{-\sigma}k^{-\sigma}$. This implies that terms $k^{-\sigma}$ and $\mu$ grow at the same rate, $\mu / \mu = -\sigma k / k = -\sigma \phi$. This, (3), (11), (14), $c = C/k$ and $g = G/K$ produce

$$\rho + (\sigma - 1)\phi = \rho - \phi - \mu / \mu = [(1 - \alpha)c^{1-\sigma} + zg^{1-\sigma}]k^{-\sigma} / \mu$$

$$= [(1 - \alpha)c^{1-\sigma} + zg^{1-\sigma}]e^\phi / (1 - \alpha)$$

$$= c + azc^{\sigma}g^{1-\sigma} / (1 - \alpha) = c + \partial g \quad (17)$$

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Differentiating this equation totally and noting (3) yield
\[ c(\phi, g) \quad \frac{\partial c}{\partial g} = g \frac{\partial c}{\partial \phi} \quad \frac{\partial c}{\partial \phi} = \frac{\sigma - 1}{1 + \sigma \delta g/c} > - \frac{1}{1 + \sigma \delta g/c} > -1 \quad (18) \]

Inserting \( \delta = 1 \) and (12) into (9) and solving for \( \phi \), we obtain
\[ \dot{k} = \phi = \pi(g, x)/[1 + im + v(m)] - c \quad (19) \]

A balanced-growth equilibrium exists in the model, because the model is proportional to the state variable \( k \). Inserting (18) into (19), differentiating totally and noting (12) and (14), we obtain the rate-of-investment function
\[ \phi = \Phi(x, i, g) \quad \text{with} \quad \Phi_x \equiv \partial \Phi/\partial x = \pi_x/[(1 + im + v)(1 + \partial c/\partial \phi)] < 0 \]
\[ \Phi_g \equiv \partial \Phi/\partial g = (\pi_x/\pi_g)\Phi_x > 0 \quad \text{and} \]
\[ \Phi_i \equiv \partial \Phi/\partial i = -[(c + \phi)m/\pi_x]\Phi_x = -\pi m \Phi_x/[(1 + im + v)\pi_x] < 0 \quad (20) \]

Because in equilibrium the consumption–capital ratio \( c \), the rate of investment \( \phi \) and the money–expenditure ratio \( m \) are kept constant, given (7), money supply \( M \) and the price level \( p \) grow at the same rate, \( \ddot{p}/p = M/M \). Hence, by increasing the quantity of money per agent, \( M \), at a fixed rate, the government can control the inflation rate \( i \equiv \dot{p}/p = M/M \).

4. PUBLIC POLICY WITH GOVERNMENT SERVICES

In this section, we assume that public services \( G \) are composed of \( \beta G \) units of the good, where constant \( \beta > 0 \) is the measure of inefficiency in the public sector. Because the agents are similar, aggregate capital is given by \( K = Jk \), where \( J \) is the number of agents. Since public expenditure \( \beta G \) is financed by taxes on \( xy \) and seigniorage \( iM/p \) from all \( J \) agents, the government’s budget constraint is \((iM/p + xy)J = \beta G \). Given (7), (12), (15), (20), \( g = G/K \) and \( K = Jk \), this constraint also reads as:
\[ Y(i, g, x, \beta) \equiv [(iM/p + xy)J - \beta G]/K = (c + \phi)im - x\pi_x - \beta g \]
\[ = \frac{\pi(g, x)im(i)}{1 + im + v(m)} - x\pi_x(g, x) - \beta g = 0 \quad (21) \]

Government expenditure \( G \) and public service intensity \( g \equiv G/K \) affect the net surplus of the government budget, \( Y \), directly as well as indirectly through the profit function \( \pi \). It is reasonable to assume that the former effect outweighs the latter. Furthermore, the economy must be on the increasing part of the Laffer curve. These assumptions take the following form:

**Assumption 1.** If the government budget is initially balanced, \( Y = 0 \), and the inflation rate \( i \) is kept constant, then a decrease in government
expenditure $g$ or an increase in the tax rate $x$ produces a budget surplus $\Upsilon > 0$, $\Upsilon_g \equiv \partial \Upsilon / \partial g < 0$ and $\Upsilon_x \equiv \partial \Upsilon / \partial x > 0$.

The government maximizes welfare by the tax rate $x$, the inflation rate $i$ and public service intensity $g \equiv G/K$, given the budget constraint (21). We can equivalently express the budget constraint (21) in terms of the tax rate and assume that the government maximizes welfare by $i$ and $g$, given this tax function. Differentiating (21) totally, and noting $\pi_x < 0$ from (12), definitions (13) and (15) as well as Assumption 1, we obtain

$$ x(i, g, \beta) \equiv \partial x / \partial \beta = g / \Upsilon_x > 0 \quad \eta + im / (1 + im + v) < 1 $$

$$ x_i \equiv \partial x / \partial i = \frac{m \pi_1}{(1 + im + v)\pi_x} \left( \frac{\varepsilon + im / (1 + im + v) - 1}{1 + im + v} \right) $$

(22)

Given (19), (20) and (22), the growth rate can then be specified as:

$$ k / k = \Phi(i, g, \beta) \equiv \Phi(x(i, g, \beta), i, g) $$

(23)

The government maximizes the representative agent’s utility (6) by $g$ and $i$, given the accumulation of capital (23) and the agent’s reaction function (18). It is equivalent to maximizing the Hamiltonian

$$ \Lambda = [(1 - \alpha)c(\phi(i, g, \beta), g)^{1-\sigma} + \alpha g^{1-\sigma}]k^{1-\sigma} / (1 - \sigma) + \lambda \phi(i, g, \beta)k $$

(24)

by $g$ and $i$, where variable $\lambda$ evolves according to

$$ \dot{\lambda} = \rho \lambda - \frac{\partial \Lambda}{\partial k} = (\rho - \phi)\lambda - [(1 - \alpha)c^{1-\sigma} + \alpha g^{1-\sigma}]k^{-\sigma} \lim_{t \to \infty} \mu ke^{-\rho t} = 0 $$

(25)

The maximization of the Hamiltonian (24) by $i$ yields

$$ \partial \Lambda / \partial i = [\lambda k^\sigma + (1 - \alpha)c^{-\sigma} \partial c / \partial \phi]k^{1-\sigma} \partial \phi / \partial i = 0 $$

(26)

Because the model of an agent contains only one state variable $k$ and is linearly homogeneous with respect to this, the system jumps immediately to the steady state in which $c, \phi, g, \partial \phi / \partial i, \partial \phi / \partial g$ and $\partial c / \partial \phi$ are constants. Given (26), this means that $\lambda$ and $k^{-\sigma}$ must grow at the same rate, $\dot{\lambda} / \lambda = -\sigma k / k = -\sigma \phi$. Inserting this into (25) and noting (17) yield

$$ \lambda k^\sigma = [(1 - \alpha)c^{1-\sigma} + \alpha g^{1-\sigma}] / [\rho + (\sigma - 1)\phi] = (1 - \alpha)c^{-\sigma} $$

(27)

Given this equation and (18), we obtain

$$ \dot{\lambda} k^\sigma + (1 - \alpha)c^{-\sigma} \partial c / \partial \phi > \lambda k^\sigma - (1 - \alpha)c^{-\sigma} = 0 $$

(28)

From this and (26) it follows that the same value of the inflation rate $i$ maximizes the Hamiltonian (24) and the growth rate (23), arg max $i \Lambda = \arg \max_i \phi(i, g, \beta)$. Hence, we obtain our first result as follows:
Proposition 2. Growth maximization is optimal monetary policy.

This proposition is the reverse of Ferreira’s (1999) main finding, which can be explained as follows. In an endogenous growth model, the congestion of public services must be proportional (see Proposition 1). This implies that private income is (even with the existence of the public sector) in fixed proportion to capital, and that the inflation rate, which is the only monetary instrument in the model, affects welfare only through the growth rate of the economy. In Ferreira's model, there is no congestion at all. Consequently, private income cannot be in fixed proportion to capital, and growth maximization is not optimal monetary policy.

Given (3), (18), (27), \( c = C/k \) and \( g = G/k \), the maximization of the Hamiltonian (24) by public service intensity \( g \) yields the first-order condition

\[
\frac{\partial \Lambda}{\partial g} = \left[ \lambda k^\sigma + (1 - \sigma)c^{-\sigma} \frac{\partial c}{\partial g} \right] k^{1-\sigma} \frac{\partial \phi}{\partial g} + 2g^{-\sigma}k^{1-\sigma} + (1 - \sigma)c^{-\sigma}k^{1-\sigma} \frac{\partial c}{\partial g} \\
= (1 + \sigma \frac{\partial c}{\partial g}) (1 - \sigma)c^{-\sigma}k^{1-\sigma} \frac{\partial \phi}{\partial g} + (\sigma + \sigma c \frac{\partial c}{\partial g}) (1 - \sigma)c^{-\sigma}k^{1-\sigma} \\
= (1 + \sigma \frac{\partial c}{\partial g}) (1 - \sigma)c^{-\sigma}k^{1-\sigma} (\sigma \frac{\partial \phi}{\partial g} + \sigma) = 0
\]

and \( \phi_g \equiv \frac{\partial \phi}{\partial g} = -\sigma \). This produces our second result:

Proposition 3. Public services must be chosen so that their marginal effect on the growth rate, \( \phi_g \), is equal to the marginal rate of substitution between consumption and public services with reversed sign, \( -\sigma \).

Assume first that public services do not yield utility directly, \( \vartheta = \sigma = 0 \), but affect welfare only through the growth rate. In such a case, welfare maximization is equivalent to growth maximization, which yields the first-order condition \( \phi_g = 0 \). If public services also yield utility directly, \( \sigma > 0 \), then the marginal rate of substitution between consumption and public services, \( \vartheta \), is positive. In such a case, public services must be increased above the level corresponding to growth maximization with \( \phi_g = 0 \), so that the growth rate will be negatively dependent on public services, \( \phi_g = -\vartheta < 0 \).

The optimal values of the government’s policy instruments \((i, g)\) are those maximizing the Hamiltonian (24), \((i^*, (g^*) (\beta)) \equiv \arg \max_{i, g} \Lambda(i, g, \beta, k)\). We assume that the government’s optimum is unique, which means the Hamiltonian \( \Lambda \) is strictly concave with respect to policy instruments \((i, g)\). Since in the government’s budget constraint (21) the measure of efficiency \( \beta \) acts as a price for public services \( G \), the optimal inflation rate rises and the optimal level of public services falls with a higher \( \beta \), by duality:

\[
di^*/d\beta > 0 \quad dg^*/d\beta < 0
\]

This result is intuitively obvious. If public services become more expensive and the tax rate is adjusted to keep the government’s budget in balance, then
the government has a tradeoff between two alternatives: a decrease in public services \( G \) and an increase in seigniorage, which yields a higher inflation rate \( i \).

5. PUBLIC POLICY WITH GOVERNMENT INVESTMENT

In this section, we assume that the government accumulates public capital from the good, and that one unit of public capital produces one unit of services \( G \). The stock of public capital can then be denoted by \( G \). We define the ratio between public investment and capital by \( s/G \). Given this and \( G = K \), the relative capital stock \( g/G \) evolves according to

\[
g = \frac{dG}{dt} = \frac{\dot{G}}{K} - (G/K) \dot{K}/K = s - g \dot{k}/k = s - g \phi
\]  

Because there is only one accumulating asset \( k \) in the private sector, after a change in the ratio \( g/G \) the private sector jumps to a new steady state and stays there. This means that the government is able to plan the accumulation of public capital taking the response of the private sector (i.e. the rate-of-investment function (20)) as given.

We assume that government investment \( \dot{G} \) is composed of \( \beta \dot{G} \) units of the good, where the constant \( \beta > 0 \) is the measure of inefficiency in the public sector. The government’s budget constraint is \( (xY + iM/p)J = \beta \dot{G} \), where \( xY \) is tax revenue, \( iM/p \) seigniorage and \( \beta \dot{G} \) public expenditure. Noting \( s = G/K \), (7), (15), (20) and \( K = \dot{J}k \), we can write this as follows:

\[
\beta s = \beta \frac{\dot{G}}{K} = \left( \frac{IM}{p} + xY \right) \frac{J}{K} = \frac{\pi(g, x) \eta m(i)}{1 + \frac{im}{m} + v(m)} - x\pi_x(g, x)
\]  

We assume that the economy is on the increasing part of the Laffer curve:

**Assumption 2.** The increase in the tax rate \( x \) keeping the inflation rate \( i \) constant creates more funds for public investment, \( \partial s/\partial x > 0 \) and \( x_i > 0 \).

The government maximizes welfare by the tax rate \( x \), the inflation rate \( i \) and the ratio of public investment to capital, \( s = G/K \), given the budget constraint (31). We can equivalently express the budget constraint (31) in terms of the tax rate and assume that the government maximizes welfare by \( i \) and \( s \), given this tax function. Differentiating (31) with respect to \( x \), \( i \) and \( s \), and using (12), (14) and (15), we obtain the function

\[
x(i, s, g, \beta) \quad x_s = \frac{\partial x}{\partial s} > 0 \quad x_\beta = \frac{\partial x}{\partial \beta} > 0 \quad \frac{im}{1 + im + v} + \eta < 1
\]

\[
\frac{\partial x}{\partial i} = \frac{m \pi}{(1 + im + v) \pi_x} \left( \frac{e + im/[(1 + im + v) - 1] + \eta}{\eta + im/[(1 + im + v) - 1]}ight)
\]  

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Inserting this function into (20), and noting (19), we obtain the growth rate as a function of policy instruments \( i \) and \( s \):

\[
\dot{k}/k = \phi(i, s, g, \beta) \equiv \Phi(x(i, s, g, \beta), i, g) \quad \text{with} \quad \phi_i = \partial \phi / \partial i \\
\phi_s \equiv \partial \phi / \partial s = \Phi_s x_s < 0 \quad \text{and} \quad \phi_g \equiv \partial \phi / \partial g
\]

(33)

The government maximizes the agent’s welfare (6) subject to the accumulation of assets (30) and (33). The Hamiltonian of this is given by

\[
\Gamma(i, s, g, k, \gamma, \varphi, \beta) \equiv \Omega(\phi(i, s, g, \beta), s, g, k, \gamma, \varphi) \\
\equiv [(1 - \alpha)c(\phi, g)^{1-\sigma} + \alpha g^{1-\sigma}]k^{1-\sigma}/(1 - \sigma) + \gamma \phi k + \varphi[s - g \phi]
\]

(34)

where the co-state variables \( \gamma \) and \( \varphi \) evolve according to

\[
\dot{\gamma} = \rho \gamma - \frac{\partial \Gamma}{\partial k} = (\rho - \phi) \gamma - [(1 - \alpha)c^{1-\sigma} + \alpha g^{1-\sigma}]k^{-\sigma} \lim_{t \to \infty} \gamma e^{-\rho t} = 0 \\
\dot{\varphi} = \rho \varphi - \frac{\partial \Omega}{\partial g} = (\rho + \phi) \varphi - \frac{\partial \Omega}{\partial \phi} \varphi_s - \left[(1 - \alpha)c^{-\sigma} \frac{\partial c}{\partial g} + \alpha g^{-\sigma}\right]k^{1-\sigma} \lim_{t \to \infty} \varphi z e^{-\rho t} = 0
\]

(35)

and where

\[
\frac{\partial \Omega}{\partial \phi} = (1 - \alpha)k^{-\sigma}c^{-\sigma} \frac{\partial c}{\partial \phi} + \varphi k - \varphi g
\]

(36)

In the steady state, the growth rate \( \dot{\phi} \), the ratios between private and public consumption and capital, \( c \) and \( g \), and the partial derivative \( \partial c / \partial \phi \) are constants. From equation (36) it then follows that in the steady state the terms \( k^{1-\sigma} \) and \( \gamma k \) must grow at the same rate. Given this, definition (19), the agent’s equilibrium condition (18) and equations (35), we obtain

\[
(1 - \sigma)\dot{\phi} = (1 - \sigma)\dot{k}/k = \dot{k}/k + \dot{\gamma}/\gamma = \rho - [(1 - \alpha)c^{1-\sigma} + \alpha g^{1-\sigma}]k^{-\sigma}/\gamma
\]

Solving \( \gamma \) from this and using (17), we obtain

\[
\gamma = [(1 - \alpha)c^{1-\sigma} + \alpha g^{1-\sigma}]k^{-\sigma}/[\rho + (\sigma - 1)\phi] = (1 - \alpha)c^{-\sigma}k^{-\sigma}
\]

(37)

Inserting this into (36) and noting (18) yield

\[
\frac{\partial \Omega}{\partial \phi} = (1 - \alpha)c^{-\sigma}k^{-\sigma}(1 + \partial c / \partial \phi) - \varphi g > -\varphi g
\]

(38)

The government maximizes the Hamiltonian (34) by public investment \( s \) and the inflation rate \( i \), which leads to the first-order conditions

\[
\partial \Gamma / \partial s = (\partial \Omega / \partial \phi) \phi_s + \varphi = 0 \\
\partial \Gamma / \partial i = (\partial \Omega / \partial \phi) \phi_i = 0
\]

(39)
Assume first that $\partial \Omega / \partial \phi = 0$. This, (38) and (39) produce $\phi = 0$ and $\partial \Omega / \partial \phi > 0$, which cannot be true. Hence, $\partial \Omega / \partial \phi \neq 0$ holds. Now assume $\partial \Omega / \partial \phi < 0$. From (33) and (39) it then follows that $0 = (\partial \Omega / \partial \phi) \phi_s + \phi > \phi$ and $\phi < 0$. Given this and (38), we obtain $\partial \Omega / \partial \phi > - \phi g > 0$, which is not true. Hence, $\partial \Omega / \partial \phi > 0$ holds. This means that the maximization of the Hamiltonian (34) by $i$ is equivalent to the maximization of the growth rate (33) by $i$: $\arg \max_i \Gamma = \arg \max_i \phi (i, s, g, \beta)$. We conclude that:

**Proposition 4.** Growth maximization is optimal monetary policy.

Inserting $\partial \Omega / \partial \phi$ from the left-hand equation in (39) into (38), we obtain

$$ (g - 1/\phi_s)\phi = (1 - \alpha)c^{-\sigma}k^{1-\sigma}(1 + \partial c / \partial \phi) \quad (40) $$

From (30) and (35) it follows that in the steady state where $c, g$ and $b$ are constants, $s$ must equal $\phi g$ and terms $\phi$ and $k^{1-\sigma}$ must grow at the same rate. Given this, (3), (18), (33), (35), (39) and (40), we obtain

$$ (1 - \sigma)\phi = (1 - \sigma)\frac{\dot{k}}{k} = \frac{\dot{\phi}}{\phi} = \rho + \phi - \frac{\partial \Omega}{\partial \phi} = \frac{\phi_s}{\phi} = \left[ \frac{\partial c}{\partial g} + \frac{c}{1 - \alpha} \left( \frac{c}{g} \right)^{\sigma} \right] (1 - \alpha)c^{-\sigma}k^{1-\sigma}/\phi $$

$$ = \rho + \phi + \frac{\phi_s}{\phi_s} - \left[ \frac{\partial c}{\partial g} + 1 \right] (1 - \alpha) \frac{k^{1-\sigma} g}{c^{\sigma}}/\phi = \rho + \phi + \frac{\phi_s}{\phi_s} - \left( g - \frac{1}{\phi_s} \right) g \quad (41) $$

and

$$ \rho + \sigma \phi - g \phi = -(\phi_s + g)/\phi_s $$

Given (33), the marginal rate of transformation between public investment $G = sk$ and private saving $k = \phi k$, i.e. the increase in $\phi$ when $s$ is marginally reduced, is $|\phi_s|$. Because $\pi$ is income per unit of capital $k$, $\pi/(1 + im + v)$ is the rate of return on private capital in terms of consumption (i.e. net of transaction and cash-reserve costs). Noting (17), (19), (33) and (41), we obtain

$$ \pi/(1 + im + v) = c + \phi = \rho + \sigma \phi - g \phi = -(\phi_s + g)/\phi_s = (\phi_s + g)/|\phi_s| $$

We summarize the rule for fiscal policy from this as follows:

**Proposition 5.** Public investment $s$ must be set so that the marginal rate of transformation between public investment and private saving is given by

$$ |\phi_s| = \frac{\phi_s + g}{\pi/(1 + im + v)} $$

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where \( \phi_g \) is the marginal effect of public services on the growth rate, \( \vartheta \) the marginal rate of substitution between consumption and public services, and \( \pi/(1 + im + v) \) the rate of return on private capital in terms of consumption.

This proposition is easily explained by comparing it to the corresponding result in Section 4. If public services \( g \) are produced directly from the good, then, as shown by Proposition 3, public services should be expanded up to the point at which the marginal revenue of public services for the economy as a whole, \( \phi_g + \vartheta \), is equal to zero. If public services are produced from public capital and public investment crowds out private saving, then the ratio of the marginal revenue of public capital, \( \phi_g + \vartheta \), to the rate of return on private capital, \( \pi/(1 + im + v) \), determines the marginal rate of transformation between public investment and private saving, \( |\phi_g| \).

Finally, we consider optimal public policy in the steady state where the ratios \( c, s \) and \( g \) are kept constant. Given (30) and (33), we obtain the steady-state condition \( s = f(i, s, g, \beta) \), which can be expressed in terms of \( g \) as \( g(i, s, \beta) \). Inserting this function and the steady-state conditions \( s = \phi g \) and (37) into the Hamiltonian (34), and noting (18) and (33), we obtain

\[
\Gamma = \chi(i, s, \beta) k^{1-\sigma},
\]

where

\[
\chi(i, s, \beta) \equiv [(1 - \alpha)c(\phi, g)^{1-\sigma} + \alpha g^{1-\sigma}] / (1 - \sigma) + (1 - \alpha)c(\phi, g)^{1-\sigma} \phi \]

with \( g = g(i, s, \beta) \) and \( \phi = \phi(i, s, g(i, s, \beta), \beta) \). In the steady state, the optimal values of the government’s policy instruments \( (i, s) \) are those maximizing the Lagrangean \( \Gamma \),

\[
(i^*, s^*(\beta) = \arg \max_{i,s} \Gamma = \arg \max_{i,s} \chi(i, s, \beta)
\]

Because the measure of efficiency \( \beta \) acts as a price for public investment \( \Gamma \) in the government’s budget (31), then in the steady state the increase in \( \beta \) raises the inflation rate and lowers public investment, by duality:

\[
di^*/d\beta > 0 \quad ds^*/d\beta < 0 \quad (42)
\]

The explanation of this is the same as for result (29).

6. OPTIMAL MONETARY POLICY

Propositions 2 and 4 imply that the growth rate \( \phi \) should be maximized by the inflation rate \( i \). Given (13), (20), (22) and (32), this maximization leads to the first-order condition

\[
\partial \phi / \partial i = \Phi_s x_i + \Phi_i = \Phi_s \{ x_i - \pi m / [(1 + im + v) \pi_x] \}
\]

\[
= [\epsilon - \eta] \left[ \eta + \frac{im}{1 + im + v} - 1 \right] \frac{m \pi \Phi_x}{(1 + im + v) \pi_x} = 0 \quad (43)
\]
Given (22) and (43), we reconstruct a result from Palokangas (1997):

**Proposition 6.** The inflation rate \( i \) should be chosen so that the elasticity of money holdings with respect to the inflation rate, \( \varepsilon \), is equal to the elasticity of the tax base with respect to the tax rate, \( \eta \).

This can be explained as follows. Since the inflation rate is equivalent to a tax on money, the elasticity of tax revenue with respect to the tax rate, \( 1 - \eta \), must be equal to the elasticity of seigniorage with respect to the inflation rate, \( 1 - \varepsilon \), which implies the Ramsey rule \( \eta = \varepsilon \).

Given (29) and (42), we obtain \( d(\dot{p}/p)/d\beta = di^*/d\beta > 0 \). This yields:

**Proposition 7.** The optimal inflation rate \( i = \dot{p}/p \) increases with inefficiency in the government sector (i.e. with a higher \( \beta \)).

If the government sector becomes more inefficient, then more seigniorage is needed to finance government expenditure, which speeds up inflation.

Finally, we briefly examine the consequences of monetary integration. Let there be two economies, labelled 1 and 2. The economies are similar to the model above, except that economy 1 has a more efficient public sector, \( 0 < \beta_1 < \beta_2 \). Because \( di^*/d\beta > 0 \) by (29) and (42), economy 1 has a lower inflation rate than economy 2:

\[
i_1 < i_2 \quad (44)
\]

Each economy can exercise fiscal and monetary policy independently of the other, and the growth rates of each may also differ.\(^5\) On the assumption that in both economies fiscal policy is chosen to maximize the welfare of the representative agent, we can define the economy-specific growth rates as:

\[
\phi^1 \equiv \{ \phi | \beta = \beta_1 \} \quad \text{fiscal policy optimized in economy 1}
\]
\[
\phi^2 \equiv \{ \phi | \beta = \beta_2 \} \quad \text{fiscal policy optimized in economy 2}
\]

According to Propositions 2 and 4, the independent monetary policy of economy \( j \) should maximize its growth rate \( \phi^j \). This yields

\[
i_j = \arg \max_i \phi^j \quad \text{for } j = 1, 2 \quad (45)
\]

Now assume that the two economies form a currency union so that a common central bank will set a common inflation rate \( i \) for them. This central bank maximizes a target \( W(\phi^1, \phi^2) \), which is a differentiable and increasing function of the growth rates of economies \( \phi^1 \) and \( \phi^2 \).

\(^5\) This property is mainly due to the assumption of a small economy and the exclusion of foreign direct investment from the model, but it helps in analysing monetary integration.
The maximization yields the first-order condition

$$\frac{dW}{di} = \frac{\partial W}{\partial \phi_1^i} \frac{\partial \phi_1^i}{\partial i} |_{i=1} + \frac{\partial W}{\partial \phi_2^i} \frac{\partial \phi_2^i}{\partial i} |_{i=1} = 0$$

Given this, the partial derivatives $[\partial \phi_1^i / \partial i]_{i=1}$ and $[\partial \phi_2^i / \partial i]_{i=1}$ must have different signs. In economy $j$ with $[\partial \phi_1^i / \partial i]_{i=1} > 0$, the inflation rate $i$ must be increased from $i$ to attain the growth-maximizing level $i_j$ with $[\partial \phi_1^i / \partial i]_{i=i_j} = 0$, and in economy $j$ with $[\partial \phi_1^i / \partial i]_{i=1} < 0$, $i$ must be decreased from $i$ to attain $i_j$. Given (44), this is true only when $i_1 < i < i_2$. We have thus obtained our final result:

**Proposition 8.** The establishment of the currency union will increase (decrease) the inflation rate in the economy with a more (less) efficient public sector, $i_1 < i < i_2$, and decrease the growth rate and welfare in both economies, $\phi^j |_{i=1} < \max_i \phi^j = \phi^j |_{i=i_j}$ for $j = 1, 2$.

The point that a currency union may force economies into a sub-optimal tax system by imposing a uniform inflation rate has already been made several times, e.g. in Canzoneri and Rogers (1990). This paper extends it to economies with endogenous growth. We show that when a currency union is going to accept new members which have very inefficient public sectors relative to the union itself, it should press these member candidates to eliminate the inefficiencies before the membership is granted, otherwise economic growth will slow down and the inflation rate in the union will increase.

### 7. CONCLUSIONS

This paper examines optimal public policy in an economy with endogenous growth and a complete government sector. The use of public services is assumed to be subject to congestion. The following results are obtained.

If congestion is proportional, i.e. if each agent assumes that it receives public services in direct proportion to its capital, then the average product of capital is constant, there is no equilibrium for capital stock, and capital can grow indefinitely. If congestion is only partial, i.e. if each agent assumes that it receives fewer public services than in proportion to its capital, then the average product of capital is decreasing, there is an equilibrium for capital stock and there is no persistent growth. This shows that endogenous growth is possible only with proportional congestion.

With persistent growth, inflation has two effects on the growth rate. First, its increase provides the government with more seigniorage and thereby helps to supply more government services and government investment. This promotes private output, private saving, capital accumulation and economic
growth. On the other hand, a higher inflation rate leads to higher transaction costs in the private sector, lower income, lower capital accumulation and slower growth. Where these two opposed effects exactly match, the inflation rate is optimal and the growth rate maximal. When congestion is proportional, the model of a private agent is linearly homogeneous with respect to capital, the monetary instrument affects welfare only through the growth rate of the economy, and growth-maximizing monetary policy also maximizes welfare. Since the inflation rate is equivalent to a tax on money, the optimal inflation rate is found by the Ramsey rule, setting the elasticity of the demand for money with respect to the inflation rate equal to the elasticity of the income tax base with respect to the income tax.

If public services are produced from goods and if they affect welfare only through growth, i.e. if the marginal rate of substitution between consumption and public services is zero, then growth-maximizing fiscal policy also maximizes welfare. The higher this marginal rate of substitution is, the more public services yield utility and the higher they must be above the growth-maximizing level. If public services are produced by accumulated public investment, then this crowds out the accumulation of private capital. In such a case, the marginal rate of transformation between public investment and private saving must be equal to the marginal revenue of public capital relative to the rate of return on private capital.

The more inefficient the public sector is, the more seigniorage is needed to finance government expenditure and the higher the inflation rate will be. If a new member with a relatively inefficient public sector is accepted into a currency union, then the inflation rate of the enlarged union will be below the optimal inflation rate of the new member but above that of the old members. Consequently, all will lose: the new member will have less seigniorage, lower public spending and slower growth, and the old members will have higher transaction costs, less income and slower growth.

The final conclusions of this paper are that if a stylized mathematical model is used to analyse the effects of public policy on economic growth, then one should pay attention to the proper microfoundations of the model. In particular, there must be some economic reason to have both inflation and the government sector in the model, and the effects of monetary policy should be examined in conjunction with fiscal policy. Although monetary integration can be welfare enhancing for reasons which are ignored in the model (e.g. stabilization policy), the results show that growth considerations can nevertheless be important in evaluating new members of a currency union.

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