

Bid Functions in Auctions and Fair Division Games: Experimental Evidence

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Abstract. *In auctions a seller offers a commodity for sale and collects the revenue. In fair division games the object is collectively owned by the group of bidders who equally share the revenue. We run an experiment in which the participants face four types of allocation games (auctions and fair division game under two price rules, first- versus second-price rule). We collect entire bid functions rather than bids for single values and investigate price and efficiency of the different trading institutions. We find that the first-price auction is more efficient than the second-price auction, whereas economic rationality assuming heterogeneous bidders suggests the opposite. Furthermore, we study the structure of individual bid functions.*

1. INTRODUCTION

Auctions and fair division games allow to allocate indivisible objects among a group of bidders. In an auction, the object is offered for sale by an outside agent (the seller) who collects the revenue himself. Auctions are widely used to solve allocation problems. Some well-known examples are the 'Aalsmeer (Dutch) Flower Auction', art auctions (e.g. at Christie's or Sotheby's), or online-auctions on the Internet (see, e.g., <http://www.ebay.com>).¹

In a fair division game, the object is collectively owned by the bidders. Accordingly, the price is equally distributed among them. Fair division games are used in conflict settlement, e.g. in case of inheritance, divorce, or when

1. For an extensive survey of the recent phenomenon of auctions taking place on the Internet see Lucking-Reiley (1999).

terminating a joint venture. In the latter case usually only the former business partners are interested in buying the firm due to private information about the future value of the enterprise. Consequently, the partners are the only bidders who, since they collectively own the firm, will split the selling price.²

We run auction and fair division experiments in which subjects submit sealed bids. The reselling values are independently and identically distributed. Furthermore, we study two different price rules: the first-price rule, i.e. the selling price of the object is the highest bid, and the second-price rule, i.e. the selling price is equal to the second highest bid.

It is important to empirically investigate bidding behavior within these games, since actual behavior might differ substantially from what is usually assumed in economic theory. Professional advice regarding the design of trading institutions should pay attention to actual bidding behavior. Systematic deviations from theoretic benchmarks may have tremendous implications for social welfare and the revenue that is raised in the different types of games. For instance, theoretical studies have shown that in the symmetric independent private-value case all auctions that determine the highest bidder as winner and lead to the same bidder participation are payoff-equivalent³ and a risk-neutral seller has no reason to prefer one versus the other. However, empirically, due to substantial differences in actual bidding behavior and/or sellers' price expectations sellers might very well have a strong preference for one or the other mechanism (see, e.g., Cox *et al.*, 1982, and for a survey Kagel, 1995). If bidding mechanisms differ in how much social welfare they generate, this can also be important, e.g. for the government or other public authorities.

While experiments usually follow the 'single bid approach' – i.e. each subject submits a single bid for one previously drawn reselling value – we employ the 'bid function approach': each subject submits a complete vector of bids (a bid function) since the reselling value is only subsequently drawn.⁴ Specifically, all subjects in the experiment develop bid functions for the four different types of games, the first- (respectively second-) price auction and the first- (respectively second-) price fair division game. A bid function specifies a bid for each of 11 possible private reselling values. After the bid functions have been submitted a reselling value is drawn randomly and independently for each subject and the game outcome is determined.

Compared to single bids the 'bid function approach' provides more information on individual bidding: we observe the bids for each reselling value in all rounds. Also, submitting a bid function might induce more consistent bidding.⁵

2. Fair division games are related to the so-called Hahn–Noll zero-revenue auctions; see Franciosi *et al.* (1993) for an experimental study.
3. For a comprehensive survey of the theoretical results see Wolfstetter (1996).
4. For other experiments in which subjects had to submit bid functions see Stelten and Buchta (1998) as well as Güth (1998).
5. On the other hand, the 'bid function approach' might have the disadvantage of 'cold' decision-making in the sense that one has to imagine several equally likely situations instead of making the 'hot' choice for a given value.

Bid Functions in Auctions and Fair Division Games

For instance, if one plans to submit a monotonic bid function, but finds one of its bids (for some reselling value) unreasonable, one may reconsider and readjust the entire bid function. This might reduce inconsistencies and the 'noise' in the data.

Our main results are that there are systematic differences between game types regarding expected price and expected efficiency. Interestingly we find that the second-price auction is less efficient than the first-price auction which is quite the opposite of what should be expected if bidders are rational and exhibit different degrees of risk aversion. We also provide an analysis of individual bidding behavior. It is shown that despite substantial variation in individual behavior, the bid functions are systematically structured.

The paper proceeds as follows: Section 2 describes the experimental games and the benchmark solutions. It also informs about the experimental procedure and payments to subjects. Sections 3 and 4 explore differences in price and efficiency between the four game types. In Section 5 we investigate the structure of individual bid functions. Section 6 concludes.

2. AUCTIONS AND FAIR DIVISION GAMES

2.1. Games and theoretical solutions

We study *sealed-bid* experiments in which a single object is to be allocated and for which each potential buyer has an *independent private value*. There are four different allocation rules to which we refer as game types (see Table 1): *First-Price Auction* (A1), *Second-Price Auction* (A2), *First-Price Fair Division Game* (F1), and *Second-Price Fair Division Game* (F2). In an auction a third party (the seller) offers an object for sale to a group of bidders and collects the price. Compared to an auction a fair division game exhibits a different ownership structure. A fair division game resembles the problem of allocating inheritance. An object is collectively owned by a group of bidders, the heirs, who submit bids. One bidder receives the object and the price is equally distributed among all bidders. The first-price rule specifies that the object is awarded to the highest bidder at a price equal to his own bid. Under the second-price rule, again, the object is awarded to the highest bidder, but the price is equal to the second highest bid.

In all four situations each bidder knows only his own evaluation of the object but not those of the other bidders. However, the probability distribution from which individual valuations are drawn is the same for all bidders and this

Table 1 The four game types

Price rule	Auction	Fair division game
Price = highest bid	A1	F1
Price = second highest bid	A2	F2

Table 2 Bid function $b_i^*(v_i)$, expected price $E(p^*)$, and expected payoffs of the equilibrium for risk-neutral bidders and the four game types

Price	Auction	Fair division game
Highest bid	$b_i^*(v_i) = \frac{n-1}{n} v_i$	$b_i^*(v_i) = \frac{n}{n+1} v_i$
	$E(p^*) = \frac{n-1}{n+1}$	$E(p^*) = \left(\frac{n}{n+1}\right)^2$
	$E(\pi_i^*(v_i)) = \frac{v_i^n}{n}$	$E(\pi_i^*(v_i)) = \frac{v_i^n}{n} + \frac{n-1}{n(n+1)}$
Second highest bid	$b_i^*(v_i) = v_i$	$b_i^*(v_i) = \frac{n}{n+1} v_i + \frac{1}{n+1}$
	$E(p^*) = \frac{n-1}{n+1}$	$E(p^*) = \frac{n^2+1}{(n+1)^2}$
	$E(\pi_i^*(v_i)) = \frac{v_i^n}{n}$	$E(\pi_i^*(v_i)) = \frac{v_i^n}{n} + \frac{n-1}{n(n+1)}$

is commonly known (symmetry). Auctions and fair division games may be analyzed as games under incomplete information.⁶ Solutions to such games may be determined according to the notion of Bayesian equilibrium.

A bidding strategy (bid function) comprises a bid for each possible valuation. Specifically, let v_i be a bidder's private value for the object to be sold, and let v_i be drawn for each player $i = 1, \dots, n$ independently from a uniform distribution on the unit interval. If all bidders are risk-neutral, the equilibrium bid function $b_i^*(v_i)$, expected equilibrium price $E(p^*)$ and expected equilibrium payoff $E(\pi_i^*)$ are as shown in Table 2. For a derivation of these results see Güth and van Damme (1986).⁷

The theoretic solutions (see Table 2) assume that values and bids are continuous variables. In experiments, however, this is usually not the case. In our experiment both, values and bids, are discrete. Riley (1989) shows for discrete values and continuous bids that pure strategy equilibria may not exist.⁸ We are not aware of theoretical studies deriving a mixed-strategy equilibrium

6. See, e.g., the seminal analysis of auctions by Vickrey (1961) as well as the work of McAfee and McMillan (1987), Maskin and Riley (1985), Myerson (1981), and Riley and Samuelson (1981).
 7. Note that expected payoffs differ between auctions and fair division games but are the same for both price rules.
 8. Nevertheless, efficiency and revenue equivalence hold for both types of auctions.

Bid Functions in Auctions and Fair Division Games

for discrete bids and discrete values; neither for first-price auctions nor for fair division games. Such an analysis would certainly be beyond the scope of our study and beyond the capabilities of experimental participants. Rather we will rely on the continuous case as a reasonable approximation of the experimental situation and use the theoretical solution as a benchmark to be compared with actual bidding.

2.2. Experimental games and procedures

In the experiment private values \tilde{v}_i did not vary continuously, but were drawn from the set

$$\tilde{V} = \{50, 60, \dots, 150\}$$

with all values $\tilde{v}_i \in \tilde{V}$ being equally likely. These values are denoted in a fictitious currency ECU (experimental currency unit) at which subjects could resell the object to the experimenter. Subjects could choose bids \tilde{b}_i as follows:⁹

$$\tilde{b}_i \in \{0, 1, \dots, 200\}$$

Within a session each subject participated in 36 consecutive games of the four different types. Nine subjects formed a session group. In each of the 36 periods they were randomly partitioned into three groups of three bidders (random matching design). The number of bidders involved in each game ($n = 3$) was commonly known, but not their identity. All subjects in all sessions played the same sequence of games. Within periods $t = 1$ to 3 they played A1 games, within $t = 4$ to 6 they played A2, in $t = 7$ to 9 the game type was F2, and in $t = 9$ to 12 it was F1. This comprised the first block of 12 games. Then they played block 2 (periods 13 to 24) and 3 (periods 25 to 36) in the same sequence as block 1. We had no strong reasons for the sequencing A1, A2, F2, F1 within each block. We merely chose this design in order to change only one single institutional aspect (price rule respectively ownership structure) at a time. Furthermore, we started with the most familiar situation, the first-price auction.

Most participants were students of economics or business administration of Humboldt University. They had been invited by leaflets to participate in an experiment announced to last about three hours. Sessions actually were about that long.

9. For ease of comparison of the empirical bids \tilde{b}_i and values \tilde{v}_i with the benchmark solution (based on continuity and risk-neutrality) all our analyses below will be done for normalized bids b_i and values v_i , i.e.

$$v_i = \frac{\tilde{v}_i - 50}{100}, \text{ respectively } b_i = \frac{\tilde{b}_i - 50}{100}$$

Accordingly the set of possible values is $V = \{0, 0.1, \dots, 1\}$.

In each game one had to submit a complete bidding strategy (bid vector) $b_i(v_i)$. Thus, they had to enter a bid for each of the 11 values $v_i \in V$. The actual value v'_i was drawn thereafter. Payments were determined according to the game rules and using the submitted bidding strategies.¹⁰ Subjects were informed on the screen about v'_i , whether or not they were buyer, about the price p at which the object was sold, and about their own payoff π_i in that game. Then the next round followed.

So, each game type applied nine times. In the first of these nine games the bid screen was blank and each subject had to enter a vector of 11 bids (one for each $v_i \in V$). In later periods the last bid vector for the same game type was displayed as default. It could be revised or submitted as it is. Of course, this may favor the status quo and may work against adjusting behavior over time. We did it for practical reasons. If subjects do not want to always adjust all bids, this saves time and helps to prevent getting bored by the task. Altogether we ran six sessions and collected 1,944 bidding strategies (54 subjects times 36 games).

Subjects' total earnings out of the 36 games ranged from 31 DEM to 96 DEM with a mean of 56 DEM (including a show-up fee of 10 DEM). In the first three sessions we used the same conversion rate for ECU (experimental currency unit) into cash for all four game types: 20 ECU = 1 DEM. Theoretically and practically this generates rather asymmetric monetary incentives for auctions compared to fair division games. Therefore, in order to induce equal expected payoffs in sessions 4 to 6 we applied a conversion rate of 3.50 ECU = 1 DEM (auctions) and 35 ECU = 1 DEM (fair division games). These are the conversion rates for which actual profits in sessions 1 to 3 would have been equal.¹¹ Essentially this implies that we had a payoff treatment: three sessions with equal conversion rate and three sessions with unequal conversion rate. Theoretically these payoff differences are irrelevant. And, since in all data analyses we did not find them being relevant, we will not discuss them any further.

3. INFLUENCE OF GAME TYPE ON EXPECTED PRICE

An interesting question in auctions is how the price at which an object is sold is influenced by the price rule. Obviously, this is quite important for the seller. Usually it will be up to him to choose the auction type, and he might want to choose that type which yields the higher price. Since the price depends on the realizations of the randomly drawn values v_i of all bidders i , we cannot simply compare the experimentally observed average prices for the two game types.

10. The strategy method obviously provides more information than collecting only one bid for a single value. But since *ex-post* only one component of the bid vector is payoff-relevant, it lowers the incentives of bidding at each single value. By restricting the set V we have tried to achieve a reasonable compromise.
11. Güth (1998) tried to guarantee equal monetary incentives by adjusting the conversion rate such that equilibrium profits were equal for $v_i = 0.5$.

Bid Functions in Auctions and Fair Division Games

Any difference in the realizations of the reselling values between the game types would induce a bias. Instead, we will determine the respective expected prices.

To do that it is a great advantage that we collected bid functions rather than single bids. This allows to determine the expected price for each group of three bidders by considering all possible combinations of reselling values. So, we consider each vector $v = (v_1, v_2, v_3)$ where v_1 to v_3 represent the values of the three bidders forming a bidder group. Since each v_i can take one of 11 values this may result in 1,331 ($= 11^3$) different vectors v . We determined the price $p(v)$ for each v . The mean of these prices represents the expected price p_j^e for the respective bidder group j . Remember that in each session each type of auction was played nine times by three groups of three randomly matched bidders. So in each session we collected 27 observations of p_j^e . Thus the six sessions yield 162 ($= 6$ times 27) observations altogether.

Figure 7 (see the Appendix) shows the cumulative distribution function (CDF) of the prices p_j^e for both auction types. The CDF for A2 is always above that one for A1 meaning that CDF(A1) first-order stochastically dominates CDF(A2).¹² Thus, the expected price is higher for A1 than for A2. A seller should clearly prefer the first-price auction. This conclusion is in line with the results, e.g. in Cox *et al.* (1982). It is confirmed by Table 3 which reports means of the prices p_j^e for each type of auction and each session (standard deviations in parentheses). The expected price is higher in A1 than in A2 for each of the six sessions. A binomial test based on the session means indicates statistical significance ($p = 0.032$, $N = 6$, two-tailed) of our finding. The test result is not influenced by the bidders' experience level.

One can justify this result by risk aversion (see the discussion in Section 5) which pushes the A1 bid functions upwards and leaves the A2 bid functions unchanged (remember that price equivalence between A1 and A2, which is a common result in auction theory, requires risk neutrality).¹³

For fair division games, Table 3 and Figure 8 (see the Appendix) report that expected price is higher in F1 than in F2. Again, this holds for all six sessions. One may argue that in fair division games differences in expected prices between these price rules are of minor importance. The price serves only to redistribute money within the group of bidders.¹⁴ Since, usually, it will be the bidders (e.g. heirs) themselves who choose the price rule, maximizing (or minimizing) the price is, in general, not a common goal of all bidders.¹⁵ Therefore, the price difference between F1 and F2 is no unambiguous criterion to solve the bidders' institutional choice problem.

12. See e.g. Laffont (1989, p. 32).

13. For a general theoretical analysis how risk aversion affects the revenue equivalent result see, e.g., Maskin and Riley (1984), Matthews (1987).

14. Actually the 'price' is just n times the compensation which the buyer pays to the others.

15. Bidders with high (low) private values who are (un)likely to buy should be interested in the F1 (F2) mechanism.

Table 3 Mean expected price (standard deviation)

Session	A1	A2	F1	F2
1	0.700 (0.07)	0.574 (0.08)	0.738 (0.10)	0.525 (0.20)
2	0.715 (0.03)	0.538 (0.12)	0.770 (0.06)	0.540 (0.10)
3	0.698 (0.04)	0.545 (0.08)	0.771 (0.10)	0.715 (0.16)
4	0.717 (0.05)	0.568 (0.07)	0.720 (0.08)	0.593 (0.13)
5	0.704 (0.03)	0.540 (0.06)	0.743 (0.07)	0.500 (0.17)
6	0.633 (0.07)	0.526 (0.07)	0.707 (0.05)	0.552 (0.06)
All	0.695 (0.06)	0.549 (0.08)	0.741 (0.08)	0.571 (0.16)

However, there may be third parties who are interested in differences in expected prices. For instance, lawyers whose fees for conducting the fair division game may depend on the price or public authorities who might want to collect taxes based on price (note that taxes can hardly depend on the net payoff to the buyer ($v_i - p$), since v_i is unobservable). Such third parties may also influence or even determine the choice of the pricing rule. So, the comparison of prices between fair division games is indeed important.

Result 1. Expected prices are higher under the first-price rule for both auctions as well as fair division games.

4. INFLUENCE OF GAME TYPE ON EXPECTED EFFICIENCY

We now look for efficiency differences between game types. An allocation is efficient if the bidder with the highest reselling value gets the object. In the empirical analysis we apply two measures of efficiency: the relative frequency of games resulting in an efficient allocation and the 'efficiency rate':

$$\text{efficiency rate} = \frac{v_i(\text{buyer})}{\max\{v_1, v_2, v_3\}}$$

where $v_i(\text{buyer})$ is the private value of the buyer. Note that the efficiency rate is bounded between 0 and 1. It is 1 if the bidder with the highest value buys the object. Similar to our above analysis of price effects we determined the expected efficiency for each game type (considering all possible combinations of reselling values).

Figure 9 (see the Appendix) shows the CDF of the observed (expected) efficiency rates for A1 versus A2. Figure 10 (see the Appendix) is the corresponding presentation for F1 versus F2. Both figures strongly suggest that the first-price rule outperforms the second-price rule. Expected efficiency is higher in A1 compared to A2 and in F1 compared to F2. This conclusion is

Bid Functions in Auctions and Fair Division Games

Table 4 Mean efficiency rates (%) and percentage of efficient allocations (in parentheses)

Session	A1	A2	F1	F2
1	97.4 (89.0)	93.3 (83.0)	92.9 (79.2)	88.9 (69.6)
2	98.4 (92.3)	94.3 (79.6)	96.9 (87.2)	90.3 (71.4)
3	98.2 (91.0)	95.9 (84.6)	96.5 (85.7)	91.5 (73.4)
4	98.3 (91.4)	90.5 (77.5)	96.9 (87.0)	91.4 (75.7)
5	99.4 (95.7)	97.1 (90.4)	96.6 (89.0)	92.5 (75.7)
6	96.5 (86.2)	95.2 (84.9)	95.1 (82.0)	96.3 (85.0)
All	98.0 (91.0)	94.4 (83.3)	95.8 (85.0)	91.8 (75.1)

confirmed by Table 4 showing the mean efficiency rate for each game type and each session, as well as the relative frequency of efficient games for each game type and each session. Expected efficiency of A1 is higher than that of A2 in all six sessions. So, the shift is statistically significant according to a binomial test ($p = 0.032$, $N = 6$, two-tailed). In particular this effect holds for inexperienced subjects. For block 1 (early periods) the mean efficiency rate is 97.0% in A1 and 91.3% in A2. For block 3 (later periods), however, mean efficiency rate is 98.6% in A1 and 95.9% in A2. Thus efficiency differences might vanish with sufficient experience.

The data for fair division games show higher expected efficiency of F1 compared to F2 in five out of six sessions. This does not allow to reject the null hypothesis ($p = 0.219$, $N = 6$, two-tailed) based on session aggregates, despite the distributions of individual decisions displayed in Figure 10 (see the Appendix). Note, however, that we have chosen a rather conservative test procedure.

We summarize:

Result 2. Expected efficiency is higher under the first-price rule than under the second-price rule. In particular it is higher for inexperienced bidders in A1 compared to A2. A social planner should prefer the first-price rule.

The efficiency loss of A2 compared to A1 is puzzling. It cannot be caused by risk aversion. Since risk aversion does not matter in A2, efficiency should be 100% for all types of (risk-averse or risk-neutral) bidders. In A1, however, heterogeneous degrees of risk aversion (like other asymmetries) may reduce efficiency.¹⁶ So, theoretically one should observe quite the opposite of what we observe here. Our finding is in line with what is reported by Kagel (1995,

16. See, for instance, Plum (1992) for a theoretical and Güth *et al.* (2001) for an experimental study of the case where private valuations of the bidders are independently drawn from distinct but commonly known distributions, one of which stochastically dominates the other.

p. 511). It is contrary to Cox *et al.* (1982). However, the latter study was rather artificial in that bidding above true value was not allowed. Furthermore, to the best of our knowledge our study provides the first between-groups statistical test of this hypothesis using independent group data.

A possible, however speculative, explanation is that the second-price rule might be more difficult to understand by boundedly rational bidders. For instance, they might be more familiar with paying their own bid in case of winning an auction rather than paying the second highest bid. This might induce noisier decisions for A2 despite the fact that, from a normative perspective, A2 is simpler¹⁷ than A1.

5. INDIVIDUAL BID FUNCTIONS

5.1. Risk-neutral equilibrium bidding

We will first check whether the data are in line with the risk-neutral equilibrium (RNE) benchmark solution as it is described in Table 2. Figures 2 to 5 (see the Appendix) show the distributions of bids for $v_i = 0.5$ for the different game types. In this case ($v_i = 0.5$) RNE predicts the following bids (see Table 2):

$$\text{A1: } b_i^* = \frac{1}{3} \quad \text{A2: } b_i^* = \frac{1}{2} \quad \text{F1: } b_i^* = \frac{3}{8} \quad \text{and} \quad \text{F2: } b_i^* = \frac{5}{8}$$

Looking at the data we find that for A1 the mode of the distribution is close to $b_i^* = \frac{1}{3}$. For A2 the mode is in fact at the theoretical prediction $b_i^* = \frac{1}{2}$. However, the modes for F1 and F2 are (roughly) at $\frac{1}{2}$ as well, even though this is not in line with RNE. Furthermore, all distributions exhibit considerable dispersion. We observe similar patterns in the distributions of bids for other reselling values $v_i \neq 0.5$, which we do not present here.

In principle, the data might be explained by a theory that combines RNE and a sufficiently high error probability ('RNE plus noise'). However, the displayed distributions are quite asymmetric around the equilibrium prediction, especially for A1, F1, and F2, suggesting a systematic deviation from RNE rather than an unbiased error. We will show below that risk aversion may cause such biased bids and may partly explain these observations.

5.2. Comparative statics

Even without the assumption that all bidders are risk-neutral, it could be that the qualitative properties of the theory regarding the effects of changes in the price rule and the ownership structure (auctions versus fair division games) are

17. A2 is incentive compatible, i.e. $b_i(v_i) = v_i$ is the only (weakly) undominated strategy for all v_i and bidders i , whereas solving A1 requires to derive the equilibrium of a Bayesian game.

Bid Functions in Auctions and Fair Division Games

Table 5 The percentage of subjects who behave in line with the comparative statics for each session and for three levels of experience (block 1 to block 3)

Comparison		A1–A2	F1–F2	A1–F1	F1–A2	A2–F2	A1–F2
Block 1	Session						
Round 1–12	1	100	78	78	67	67	78
	2	67	56	78	56	56	67
	3	67	78	78	44	89	78
	4	89	67	67	78	78	89
	5	78	44	33	78	11	33
	6	89	89	100	56	78	100
Block 2	Session						
Round 13–24	1	89	67	67	67	44	78
	2	78	67	89	78	56	67
	3	89	100	56	78	56	100
	4	78	78	78	78	56	89
	5	100	67	78	78	33	78
	6	89	78	89	67	44	89
Block 3	Session						
Round 25–36	1	100	78	67	78	44	78
	2	78	67	78	78	56	56
	3	89	89	56	78	56	78
	4	89	78	78	78	56	89
	5	100	89	89	89	67	100
	6	89	67	78	67	67	100

still applicable.¹⁸ For risk-neutral agents the equilibrium bid functions for the four game types satisfy the following relations (see also Figure 1a in the Appendix):

$$b_{i,A1}^*(v_i) \leq b_{i,F1}^*(v_i) \leq b_{i,A2}^*(v_i) \leq b_{i,F2}^*(v_i) \quad (1)$$

for all values v_i and where the inequalities are strict if one neglects $v_i = 0$ and $v_i = 1$. We refer to this as the ‘comparative statics’ (of game types).

Table 5 reports the percentage of subjects who behave in line with the comparative statics for each session and for three levels of experience (block 1 to block 3). The head row shows the respective two game types under consideration. A subject is considered in line with the comparative statics if its average bids (over all three games in the respective block) satisfy the theoretical order. For example, in block 1 (low experience) 100% of subjects in session 1 chose strictly lower mean bids in A1 than in A2 (first column). Seventy-eight

18. For instance, Kagel and Levin (1993) observe in sealed-bid private value auctions successful comparative static predictions of varying the price rule and the number of bidders, contrary to experiments in the common value case (see Dyer *et al.*, 1989).

per cent of the subjects in session 1 chose strictly lower mean bids in F1 than F2 (second column). The other columns can be read accordingly. Looking down the first column one finds that the percentages vary between sessions and for different levels of experience, but they are 56% (five out of nine subjects) or higher in almost all cases. Thus in most cases more than half of the subjects chose bids in line with the comparative statics. We obtain similar results for the other comparisons in Table 5.

One may derive a statistical test of the comparative statics hypothesis as follows: if at least five out of nine subjects (56%) obey the comparative static prediction we consider the respective session in line with the theory. Comparing the two auctions, A1–A2 (first column), we find that this is the case for all six sessions. So a binomial test rejects the null hypothesis (no systematic shift) in favor of the comparative statics prediction ($p = 0.016$, one-tailed, $N = 6$). The same holds for a comparison of the two fair division games, F1–F2, for experienced bidders (block 3). Similar tests can be derived for A1–F1, F1–A2, A2–F2, and A1–F2. For experienced bidders the tests support the theory for all comparisons but A2–F2 ($p = 0.109$). The individual data analysis supports the finding from the regression analyses shown in Figure 1b (see the Appendix). Thus, in sum we conclude that the observed bid functions satisfy the comparative statics derived from the benchmark case assuming continuous bids and values as well as risk-neutral bidders.

5.3. Slope and curvature of bid functions

In the following we will investigate the slopes and curvatures of the observed bid functions. As a weak requirement of rational bidding (compared to RNE bidding) one might expect the bid functions to be upward sloping (i.e. increasing in v_i). We estimate the following regression model:

$$b_i = \alpha_0 + \alpha_1 v_i^{\text{low}} + \alpha_2 v_i^{\text{high}}$$

with

$$v_i^{\text{low}} = \begin{cases} v_i & \text{if } v_i < 0.5 \\ 0.5 & \text{otherwise} \end{cases}$$

and

$$v_i^{\text{high}} = \begin{cases} v_i - 0.5 & \text{if } v_i \geq 0.5 \\ 0 & \text{otherwise} \end{cases}$$

Thus, we fit a piecewise-linear regression line to the data to yield approximations of convex or concave individual bid functions allowing for a kink at $v_i = 0.5$. If a bid function is perfectly linear, the slope parameters α_1 and α_2 are equal. The regression model is estimated for each individual ($i = 1$ to 54) and each period ($t = 1$ to 36) separately, giving a total of 1,944 estimated individual

Bid Functions in Auctions and Fair Division Games

Table 6 Slope and curvature of the estimated individual bid functions

Game type	Strictly increasing				Weakly decreasing
	Concave	Linear	Convex	All	
A1	32.3%	46.3%	20.4%	99.0%	1.0%
A2	21.0%	53.9%	23.0%	97.9%	2.1%
F1	33.5%	46.3%	15.0%	94.9%	5.1%
F2	36.0%	42.4%	19.1%	97.5%	2.5%
All	30.7%	47.2%	19.4%	97.3%	2.7%

Note: 100% = 1,944 estimated individual bid functions.

bid functions. Based on these analyses bid functions are classified as strictly increasing (and furthermore as concave, linear, or convex) or weakly decreasing (see Table 6).

A total of 1,892 ($\approx 97\%$) of the bid functions are strictly increasing, i.e. $\alpha_1 > 0$ and $\alpha_2 > 0$. To determine the curvature of the bid functions we computed the difference between the two slope coefficients $\Delta \equiv \alpha_2 - \alpha_1$. We consider a bid function as linear if $|\Delta| \leq \pm 0.025$.¹⁹ If $\Delta < -0.025$ ($\Delta > +0.025$), the respective bid function is classified as concave (convex). Owing to these criteria about 47% of the estimated functions are linear, 31% are concave, and 19% are convex.²⁰ Overall, the fit of the piecewise-linear regression model is remarkable: 1,867 estimated functions (94%) exhibit R^2 values above 90%. We conclude that despite great differences in individual behavior most bid functions exhibit natural and regular structures: almost all are strictly increasing and can be approximated rather well by piecewise-linear functions; many are in fact linear.

5.4. Bidding areas

Since the data reveal a substantial variation in bidding behavior, an interesting question arises: what type of behavior can be rationalized by assuming general forms of heterogeneous risk attitudes? One can show that if bidders exhibit different degrees of (weak) risk aversion,²¹ rational bids should lie within a

19. Of course, we could run formal tests for the theoretical restrictions imposed on α_1 and α_2 . We prefer to simply report some descriptive measures since the usual assumptions for statistical testing are hardly satisfied here. Note that each regression is based on 11 data points drawn from a single subject. Furthermore we estimate 36 regressions for the same subject.
20. The criterion for linearity which we chose is ad hoc. It could have been set more (or less) restrictive. About 11% of the bid functions were exactly linear ($|\Delta| = 0$). For $|\Delta| \leq \pm 0.05$ the proportion of linear functions increases to 54.5%.
21. This includes risk neutrality as a boundary case. Risk neutrality might be justified by the comparatively small effects of experimental earnings on income (see Rabin, 2000).

bidding area that is bounded above and below by two bid functions: the RNE (of the respective game type) and the ‘true value’ bid function. This is trivial for A2 since ‘true value’ bidding (= RNE) is a dominant strategy.²² To see why those boundaries hold in A1 note that:

- (1) Bids $b_i(v_i) \geq v_i$ are weakly dominated, and can never be rational. By true value bidding ($b_i = v_i$) a bidder gets zero for sure. Bidding below v_i induces a positive payoff in case i wins the auction and a zero payoff in case i does not win.
- (2) Any further reduction in b_i increases the payoff in case i wins, but at the same time reduces the probability of winning. In general, a risk-neutral bidder will therefore choose a lower bid than a risk-averse bidder, given that both face the same (possibly mixed) population of bidders.
- (3) RNE bidding maximizes bidder i 's expected payoff if all bidders are risk-neutral (and therefore submit RNE equilibrium bids). If another bidder j bids above RNE, a risk-neutral bidder i should bid above RNE as well.

Together, statements (1) to (3) establish our conclusion that the area for rational bidding in A1 is bounded below by RNE and bounded above by ‘true value’ bidding. Similar statements can be given for fair division games. We conclude that with heterogeneous degrees of risk aversion the rational bidding areas are:

- bounded below by RNE and above by ‘true value’ bidding for A1, A2, and F1;
- bounded below by ‘true value’ bidding and above by RNE for F2.

Compared to point predictions that may be derived from specific models of risk attitudes such set valued predictions seem much more in line with the fact that risk attitudes can vary between bidders and are unknown to others.

Figure 6 (see the Appendix) displays the prediction areas for the different game types and reports the percentage of bids that lie within the respective

Table 7 Predictive success of bidding areas

Game type	Predictive success			
	Block 1	Block 2	Block 3	All
A1	70.0%	78.8%	82.5%	77.1%
A2	37.3%	43.3%	48.7%	43.1%
F1	64.8%	66.2%	62.1%	64.4%
F2	36.3%	43.7%	49.4%	43.1%

22. Thus, for A2 the ‘bidding area’ is just a line ($b_i(v_i) = v_i$).

Bid Functions in Auctions and Fair Division Games

area. Further details are provided in Table 7.²³ Thus, in A1 (F1, F2) 77% (64%, 43%) of all bids cluster within the bidding area and may therefore be rationalized by the assumption of heterogeneous risk aversion. In A2 the prediction area covers 43% of the bids.²⁴ Except for F1 the frequency of bids inside the prediction areas increase with experience. These statistics suggest two things: the economic theory of heterogeneous (weakly) risk-averse, rational bidding organizes a substantial part of the data, but at the same time there is a substantial part of the data that cannot be explained by whatever form of risk aversion.

To what extent bidding behavior can be explained by risk aversion is controversial among experimental economists. Cox *et al.* (1988, 1992) develop a model with heterogeneous bidders exhibiting constant relative risk aversion (CRRAM) and believe that it can organize the data rather well. Others, for instance, Harrison (1989, 1992) and Kagel and Roth (1992), are more skeptical and provide evidence that cannot easily be reconciled with CRRAM.²⁵

6 . CONCLUDING REMARKS

We reported evidence from an experiment on different types of allocation games: auctions versus fair division games under the first-, respectively the second-price rule. We applied the 'bid function approach' instead of the 'single bid approach'. The latter is more usual in auction experiments and it is more realistic in the sense that in real-world auctions and fair division games participants submit bids for their individual (single) value. However, theoretical models of bidding behavior, e.g. risk-neutral equilibrium bidding, rely on the assumption that bidding is structured according to underlying individual bid functions. Therefore we directly asked our experimental participants to provide bid functions in order to find out how these are structured and to investigate price and efficiency properties of the different allocation games given this decision format.²⁶

23. We allow for a tolerance of ± 0.025 at each boundary of the prediction areas. Specifically, we consider $b_i^* - 0.025$ and $v_i + 0.025$ for A1, A2, and F1 as well as $b_i^* + 0.025$ and $v_i - 0.025$ for F2. This allows for some errors in bidding and seems reasonable especially in the case of A2 where the theoretical prediction area is just a line. For symmetry we apply this rule for the other three game types as well.
24. The share of bidders who bid exactly according to the dominant strategy (for all values and at least in one out of three rounds of the respective block) is 24% within block 1 and 20% within blocks 2 and 3.
25. For more details concerning the active debate on this topic see the December 1992 issue of *American Economic Review*.
26. The experiment also allows for studying learning behavior within and across different allocation rules. We investigate learning effects in a companion paper (Güth *et al.*, 1999). For instance, we show that learning does not drive bidding towards the benchmark solution and that directional learning theory (see Selten and Buchta, 1998) offers a partial explanation for bid changes.

Our main results are that differences in the applied price rule (first- versus second-price) leads to significant differences in expected price and expected efficiency of auctions and fair division games (results 1 and 2). We provide robust statistical tests supporting our findings. Comparing A1 and A2 we find that both expected price and expected efficiency are higher for A1. Consequently, A1 should be preferred to A2 from the perspective of a risk-neutral seller (price) as well as the perspective of a social planner (efficiency). The higher efficiency of A1 is puzzling. Namely, if bidders are rational and heterogeneously risk-averse, one should observe the opposite effect, that A2 is more efficient than A1. An explanation for our finding could be that bidders are more familiar with the first-price rule. In many real-world auctions, bidders pay their own bid. This holds for Dutch auctions and sealed-bid public tenders. Note that even though the English auction is strategically equivalent to the second-price sealed-bid auction, in the former auction bidders pay their own bids. Being unfamiliar with the second-price rule might lead to less systematic, more erratic bidding which tends to reduce efficiency. This explanation is in line with the finding that differences in efficiency are reduced by experience.²⁷

Comparing F1 and F2 the result is again a higher expected price and higher expected efficiency for the first-price rule, but only the price effect is statistically significant. While F1 is in our view a natural trading institution for the allocation of inheritance, F2 might seem rather artificial. However, since one might wonder about using such a mechanism, it is nonetheless important to study bidding behavior under this rule.

The analysis of individual bid functions is mainly descriptive rather than a theory test. While it shows that simple equilibrium models fail to predict behavior, it also shows that more general economic concepts are useful in organizing the data. We found that the (RNE) benchmark solution for the continuous case (bids and values) with risk-neutral bidders cannot explain the data very well. Nevertheless, the individual bid functions are highly structured. Almost all are monotonically increasing, which may be regarded as a weak requirement of (boundedly) rational bidding. They can be approximated quite well by a kinked linear bid function. Many bid functions are indeed linear.

The distributions of bids show large dispersions, and there are substantial individual differences in bidding behavior, which persist even when bidders gain experience. Given the observed variation in individual bidding we asked what kinds of bids can be rationalized allowing for rather general and heterogeneous levels of risk aversion (with risk neutrality as a boundary case). Based on these theoretical assumptions about bidding behavior we identified bidding areas for each allocation game and found that between 43% (A2, F2) and 77% (A1) of all bids cluster within the respective areas. Furthermore,

27. By asking each bidder to participate in all four game types (within-subject design) we controlled for heterogeneity between treatment groups (see Gächter and Königstein, 2002). To further substantiate our findings, future studies on the impact of institutional changes might apply a between-subject design.

Bid Functions in Auctions and Fair Division Games

comparing across different allocation games the bid functions obey the comparative static predictions derived from the RNE benchmark solution. Thus, theoretical analyses of bidding behavior are useful in predicting bidding behavior, even though there is a lot left to be explained by further studies.

APPENDIX

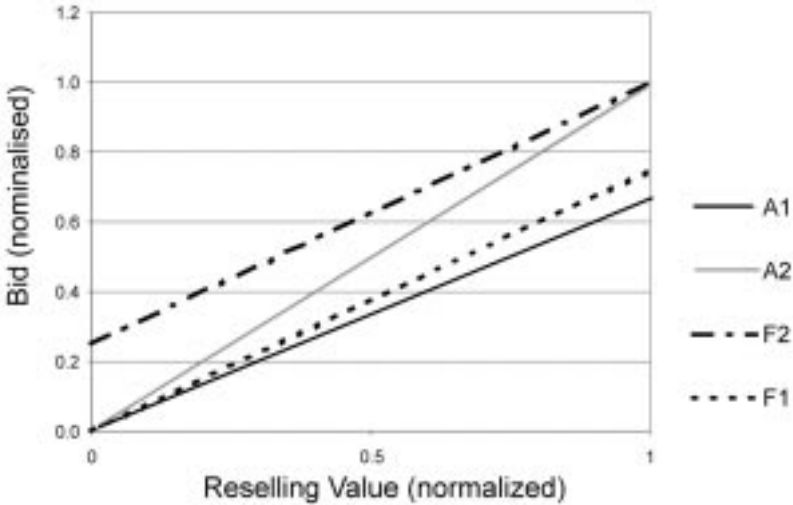


Figure 1a RNE bid functions for all game types

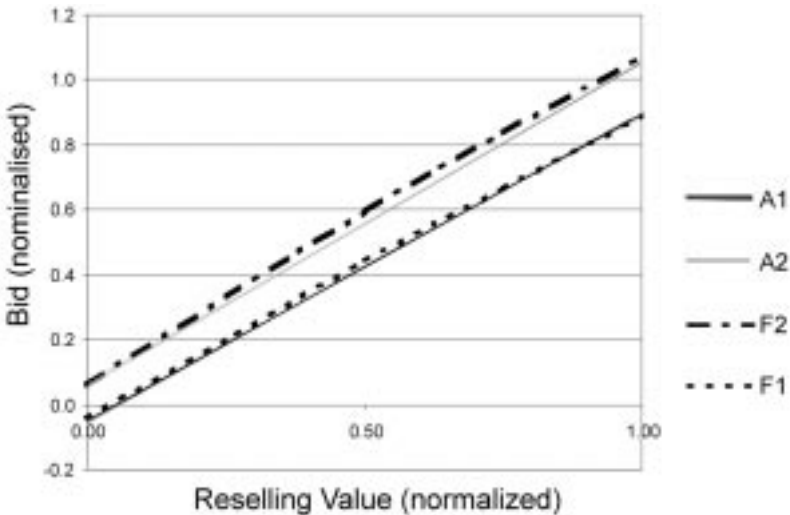


Figure 1b Estimate aggregate bid functions

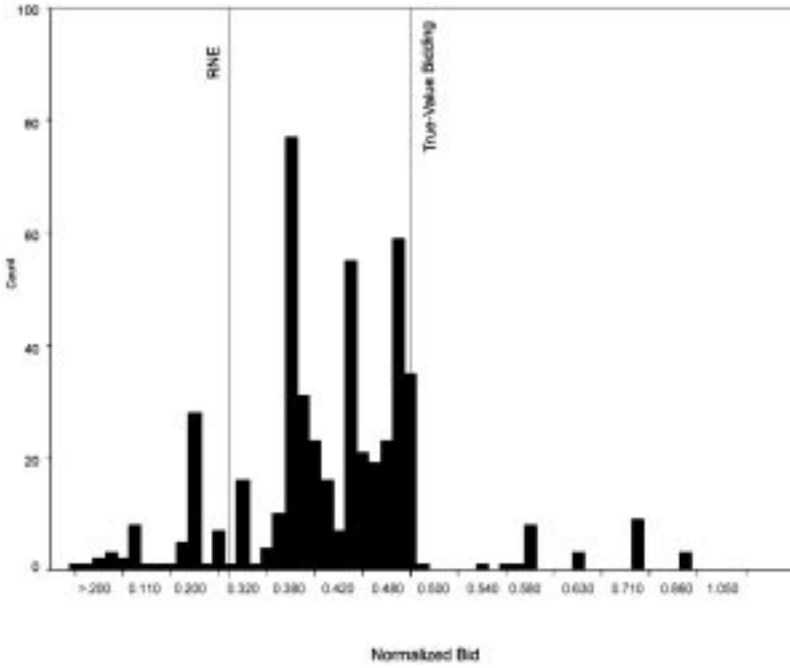


Figure 2 Distribution of bids for $v = 0.5$ (first-price auction)

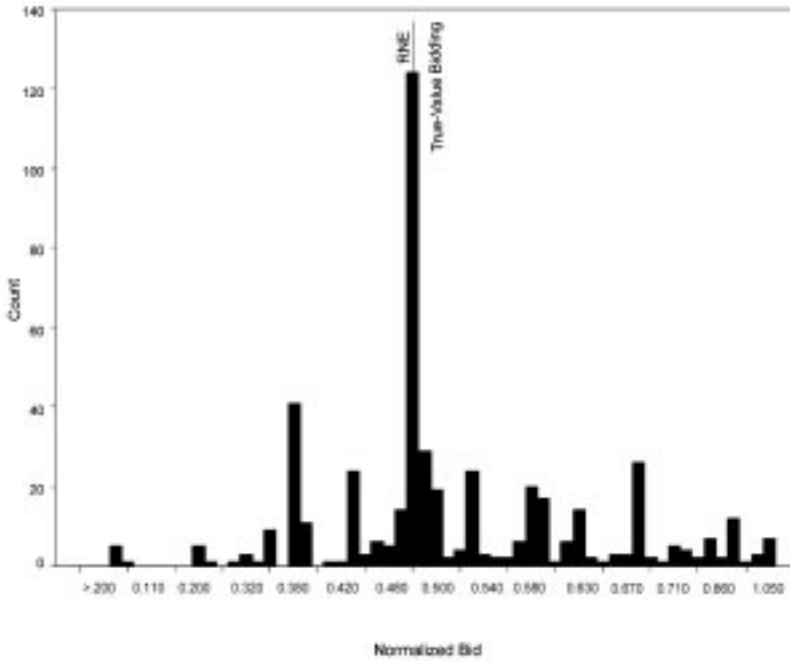


Figure 3 Distribution of bids for $v = 0.5$ (second-price auction)

Bid Functions in Auctions and Fair Division Games

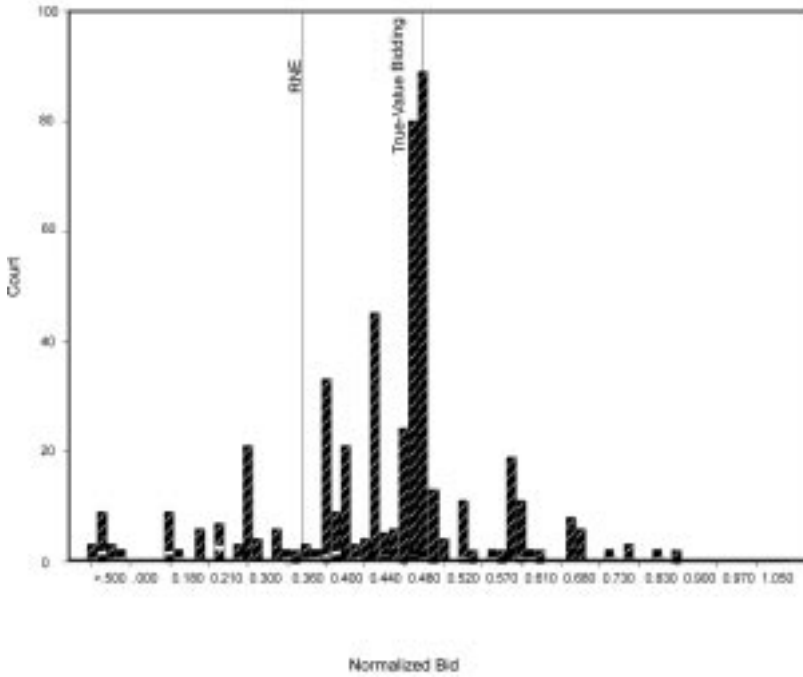


Figure 4 Distribution of bids for $v = 0.5$ (first-price fair division game)

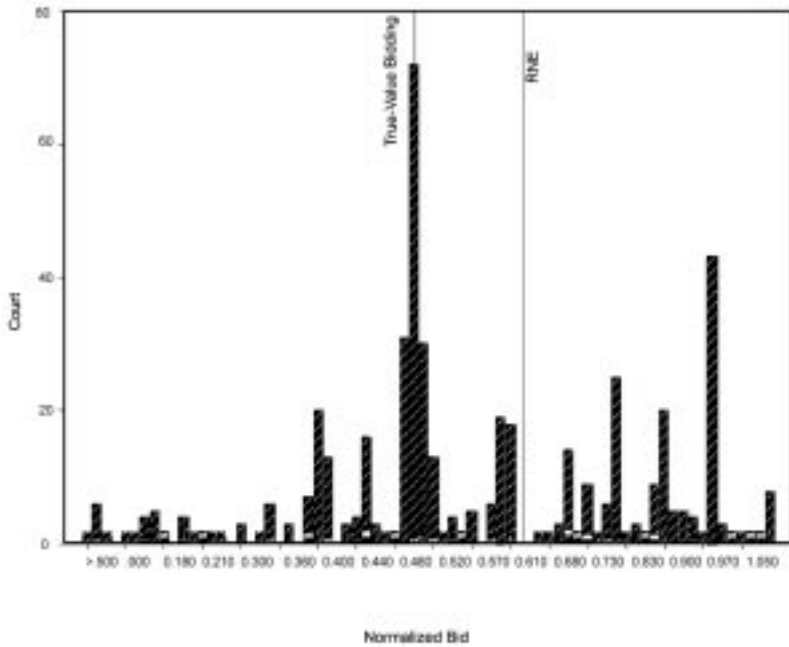


Figure 5 Distribution of bids for $v = 0.5$ (second-price fair division game)

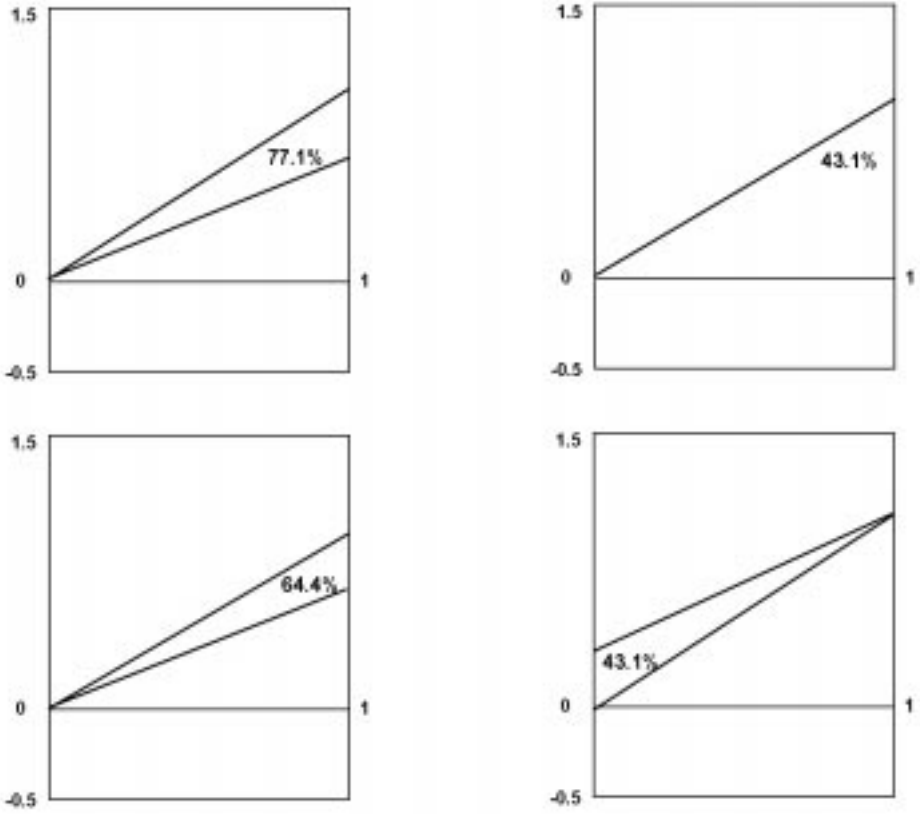


Figure 6 Prediction areas for all four cases

Bid Functions in Auctions and Fair Division Games

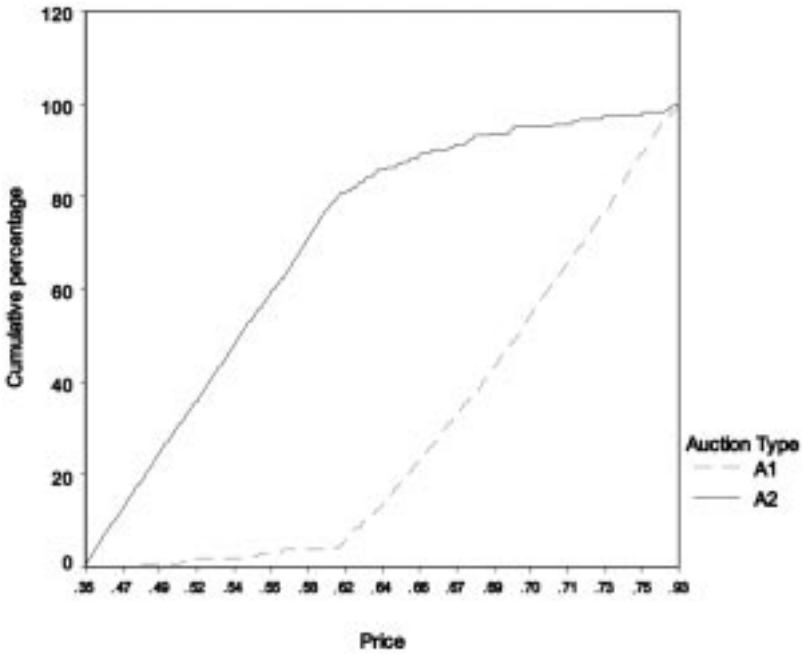


Figure 7 Cumulative distributions of the prices for both auction types

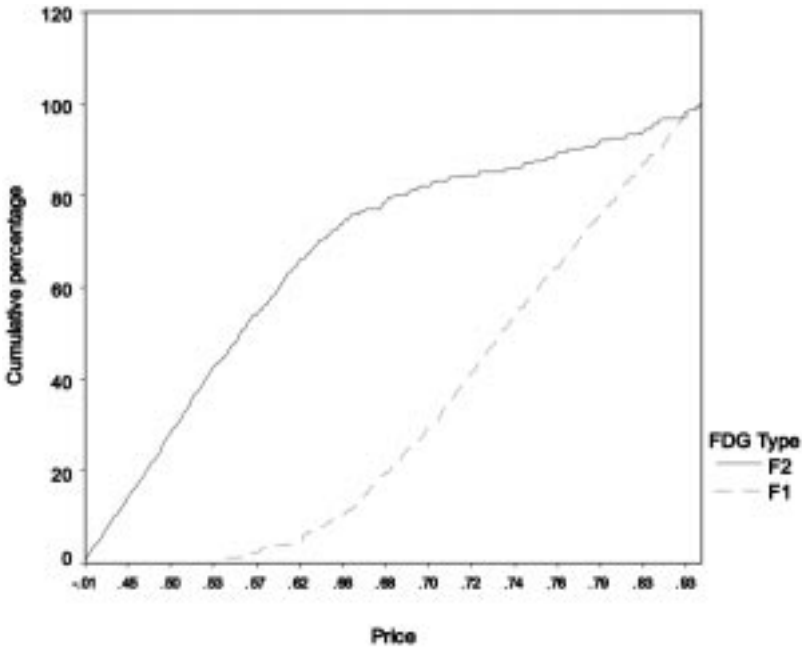


Figure 8 Cumulative distributions of the prices for both fair division game types

Note: Negative prices are admissible since our analysis is based on normalized bids b_i and values v_i .

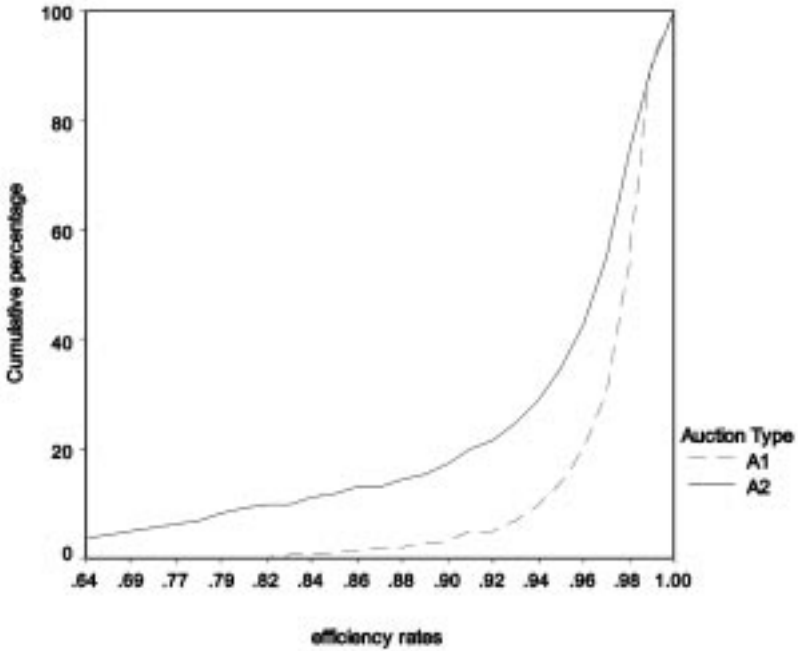


Figure 9 Cumulative distribution function of the observed efficiency rate for both auction types

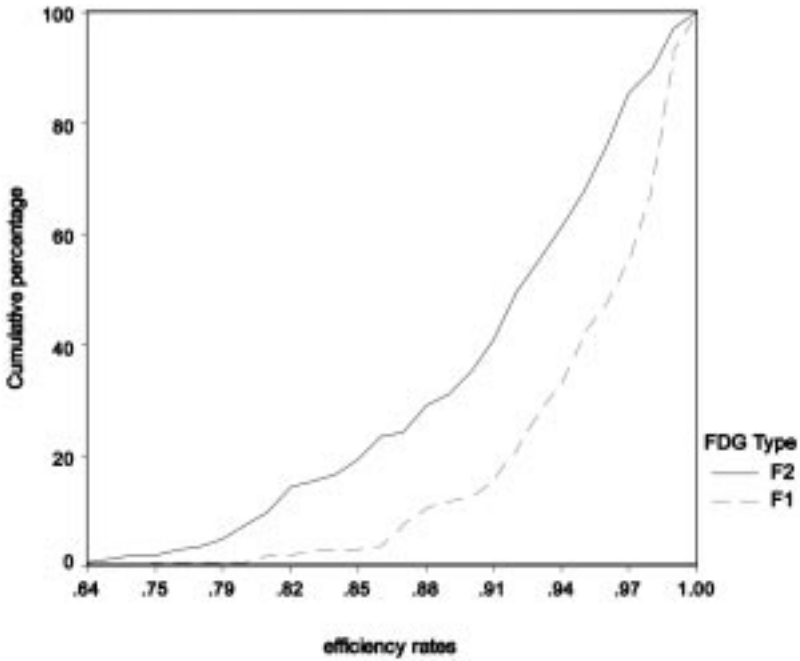


Figure 10 Cumulative distribution function of the observed efficiency rate for both fair division games

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